“Day-ahead electricity markets”

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As part of the overall context...

By the way... **why is this called a day-ahead market?**
Learning objectives

Through this lecture and additional study material, it is aimed for the students to be able to:

1. Describe an electricity pool and its auction mechanism
2. Model and solve day-ahead market clearing
3. Understand differences between zonal and nodal pricing
4. Calculate revenues and payments of market players under different settlement methods
Outline

1. Different types of electricity exchanges
   - bilateral contracts
   - auctions
   - market-clearing algorithms

2. Network effects
   - zonal pricing
   - splitting and difference between system and area price
   - extension to nodal pricing

3. Different types of market products
   - in theory
   - in practice
Different types of electricity exchanges
Bilateral contracts are for a direct exchange of power between a buyer and a seller. They may both be producers and/or consumers. Most likely a broker is involved. Eventually, the system operator is informed about the trades that occurred.
Types of bilateral trading

- **Customized long-term contracts:**
  - very flexible contracts (basically, you can try to negotiate whatever you want)
  - private transactions (conditions are fully unknown to others)
  - large transactions costs
  - large amounts of energy, over long periods of times

- **Over the counter (OTC) trading:**
  - standard contracts
  - lower transactions costs
  - typically, smaller amount and short lead times

[Note: To be discussed further in Lecture 2 when introducing intra-day markets]

- **Electronic trading** (already leaning towards the pool concept...):
  - Based on electronic platform that consistently match supply and offer bids
  - virtually no transactions costs
  - very fast, therefore allowing trading “until the last second”

[Note: To be discussed further in Lecture 5 when introducing intra-day markets]
Placing it into perspective...

- Bilateral trading may be interesting...
- but the pool provides a much more centralized form of system management, which seems to be increasingly preferred in Europe for day-ahead markets.

**Example:**

- Nord Pool Spot is the Europe’s largest power market
- 505 TWh of energy traded in 2016
- Nordic and Baltic day-ahead auction Elspot represents 391 TWh of energy traded
- Average system price of 26.91€
- In Elspot: 380 buyers/sellers - >2000 orders a day

- Let us focus on pools and auctions for now...
Auctions in an electricity pool

- All generation bids and consumption offers are placed at the same time.
- No-one knows about others’ bids and offers.
- A centralized market-clearing algorithm decides about bids and offers that are retained.
- Eventually, the system operator is informed about the trades that occurred.
An example auction setup

- **Deadline for offers**: 29th of January, 12:00 - **Delivery period**: 30th of January, 11:00-12:00

- Supply and demand offers include:

**Demand**: (for a total of 1065 MWh)

<table>
<thead>
<tr>
<th>Company</th>
<th>Supply/Demand</th>
<th>id</th>
<th>Amount (MWh)</th>
<th>Price (€/MWh)</th>
</tr>
</thead>
<tbody>
<tr>
<td>CleanRetail</td>
<td>Demand</td>
<td>$D_1$</td>
<td>250</td>
<td>200</td>
</tr>
<tr>
<td>El4You</td>
<td>Demand</td>
<td>$D_2$</td>
<td>300</td>
<td>110</td>
</tr>
<tr>
<td>EVcharge</td>
<td>Demand</td>
<td>$D_3$</td>
<td>120</td>
<td>100</td>
</tr>
<tr>
<td>QualiWatt</td>
<td>Demand</td>
<td>$D_4$</td>
<td>80</td>
<td>90</td>
</tr>
<tr>
<td>IntelliWatt</td>
<td>Demand</td>
<td>$D_5$</td>
<td>40</td>
<td>85</td>
</tr>
<tr>
<td>El4You</td>
<td>Demand</td>
<td>$D_6$</td>
<td>70</td>
<td>75</td>
</tr>
<tr>
<td>CleanRetail</td>
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<td>$D_7$</td>
<td>60</td>
<td>65</td>
</tr>
<tr>
<td>IntelliWatt</td>
<td>Demand</td>
<td>$D_8$</td>
<td>45</td>
<td>40</td>
</tr>
<tr>
<td>QualiWatt</td>
<td>Demand</td>
<td>$D_9$</td>
<td>30</td>
<td>38</td>
</tr>
<tr>
<td>IntelliWatt</td>
<td>Demand</td>
<td>$D_{10}$</td>
<td>35</td>
<td>31</td>
</tr>
<tr>
<td>CleanRetail</td>
<td>Demand</td>
<td>$D_{11}$</td>
<td>25</td>
<td>24</td>
</tr>
<tr>
<td>El4You</td>
<td>Demand</td>
<td>$D_{12}$</td>
<td>10</td>
<td>16</td>
</tr>
</tbody>
</table>
An example auction setup

Supply: (for a total of 1435 MWh)

<table>
<thead>
<tr>
<th>Company</th>
<th>Supply/Demand</th>
<th>id</th>
<th>Amount (MWh)</th>
<th>Price (€/MWh)</th>
</tr>
</thead>
<tbody>
<tr>
<td>RT® Supply</td>
<td>Supply</td>
<td>G₁</td>
<td>120</td>
<td>0</td>
</tr>
<tr>
<td>WeTrustInWind Supply</td>
<td>Supply</td>
<td>G₂</td>
<td>50</td>
<td>0</td>
</tr>
<tr>
<td>BlueHydro Supply</td>
<td>Supply</td>
<td>G₃</td>
<td>200</td>
<td>15</td>
</tr>
<tr>
<td>RT® Supply</td>
<td>Supply</td>
<td>G₄</td>
<td>400</td>
<td>30</td>
</tr>
<tr>
<td>KøbenhavnCHP Supply</td>
<td>Supply</td>
<td>G₅</td>
<td>60</td>
<td>32.5</td>
</tr>
<tr>
<td>KøbenhavnCHP Supply</td>
<td>Supply</td>
<td>G₆</td>
<td>50</td>
<td>34</td>
</tr>
<tr>
<td>KøbenhavnCHP Supply</td>
<td>Supply</td>
<td>G₇</td>
<td>60</td>
<td>36</td>
</tr>
<tr>
<td>DirtyPower Supply</td>
<td>Supply</td>
<td>G₈</td>
<td>100</td>
<td>37.5</td>
</tr>
<tr>
<td>DirtyPower Supply</td>
<td>Supply</td>
<td>G₉</td>
<td>70</td>
<td>39</td>
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<tr>
<td>DirtyPower Supply</td>
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<td>70</td>
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<tr>
<td>RT® Supply</td>
<td>Supply</td>
<td>G₁₂</td>
<td>45</td>
<td>70</td>
</tr>
<tr>
<td>SafePeak Supply</td>
<td>Supply</td>
<td>G₁₃</td>
<td>50</td>
<td>100</td>
</tr>
<tr>
<td>SafePeak Supply</td>
<td>Supply</td>
<td>G₁₄</td>
<td>60</td>
<td>150</td>
</tr>
<tr>
<td>SafePeak Supply</td>
<td>Supply</td>
<td>G₁₅</td>
<td>50</td>
<td>200</td>
</tr>
</tbody>
</table>

That is a lot of offers to match... but how?
Merit order and equilibrium

- Consumption offers are ranked in *decreasing price order*
- Supply offers are ranked in *increasing price order*
- This defines the **merit order**
- A “magic” point appears: the equilibrium point between supply and demand...
Social welfare and its maximization

- **Social welfare is defined as the area between consumption and generation**

- The equilibrium point is that which allows to maximize social welfare

- Why?
  - Any buyer is to pay at most what he was ready to pay
  - Any seller will get at minimum the price he was ready to sell for
As an optimization problem

- On the **supply side**:
  - Set of offers: $\mathcal{L}_G = \{G_i, \ i = 1, \ldots, N_G\}$
  - Maximum quantity for offer $G_i$: $P_i^G$
  - Price for offer $G_i$: $\lambda_i^G$
  - **Generation level to optimize social welfare**: $\{y_i^G, \ i = 1, \ldots, N_G\}, \ 0 \leq y_i^G \leq P_i^G$

- On the **demand side**:
  - Set of offers: $\mathcal{L}_D = \{D_i, \ i = 1, \ldots, N_D\}$
  - Maximum quantity for offer $D_i$: $P_i^D$
  - Price for offer $D_i$: $\lambda_i^D$
  - **Consumption level optimizing social welfare**: $\{y_i^D, \ i = 1, \ldots, N_D\}, \ 0 \leq y_i^D \leq P_i^D$

- The social welfare maximization problem can be written as:

\[
\begin{align*}
\max \quad & \sum_i \lambda_i^D y_i^D - \sum_j \lambda_j^G y_j^G \\
\text{subject to} \quad & \sum_j y_j^G - \sum_i y_i^D = 0 \\
& 0 \leq y_i^D \leq P_i^D, \ i = 1, \ldots, N_D \\
& 0 \leq y_j^G \leq P_j^G, \ j = 1, \ldots, N_G
\end{align*}
\]
It is a simple linear program!

- One recognize a so-called **Linear Program** (LP, here in a compact form):

\[
\begin{align*}
\min_y & \quad c^T y \\
\text{subject to} & \quad Ay \leq b \\
& \quad A_{eq}y = b_{eq} \\
& \quad y \geq 0
\end{align*}
\]

[Note: any such optimization problem can be written as a maximization or minimization problem]

- LP problems can be readily solved in
  - **Matlab**, for instance with the function linprog,
  - **R**, with the library/function lp_solve,
  - and also obviously in **GAMS**

- But, for e.g. R and Matlab, you need to know how to build relevant vectors and matrices

- And, the solution \( y^* \) will *only* give you the list of offers (supply and demand) accepted...
Vector and matrices in the objective function

The vector $y$ of optimization variables $c$ of weights in the objective function are constructed as

$$y = \begin{bmatrix} y^G_1 \\ y^G_2 \\ \vdots \\ y^G_{N_G} \\ y^D_1 \\ y^D_2 \\ \vdots \\ y^D_{N_D} \end{bmatrix}, \quad y \in \mathbb{R}^{(N_G+N_D)}$$

$$c = \begin{bmatrix} \lambda^G_1 \\ \lambda^G_2 \\ \vdots \\ \lambda^G_{N_G} \\ -\lambda^D_1 \\ -\lambda^D_2 \\ \vdots \\ -\lambda^D_{N_D} \end{bmatrix}, \quad c \in \mathbb{R}^{(N_G+N_D)}$$
Vector and matrices defining constraints

- For the equality constraint (balance of generation and consumption):

\[ A_{eq} = [1 \ldots 1 \ -1 \ldots -1], \quad A_{eq} \in \mathbb{R}^{(N_G+N_D)}, \quad b_{eq} = 0 \]

- For the inequality constraint (i.e., generation and consumption levels within limits):

\[
A = \begin{bmatrix}
1 & 1 & 0 \\
1 & 0 & 1 \\
0 & \ddots & 1
\end{bmatrix}, \quad b = \begin{bmatrix}
P^G_1 \\
P^G_2 \\
\vdots \\
P^G_{N_G} \\
P^D_1 \\
P^D_2 \\
\vdots \\
P^D_{N_D}
\end{bmatrix},
\]

with \( A \in \mathbb{R}^{(N_G+N_D) \times (N_G+N_D)} \) and \( b \in \mathbb{R}^{(N_G+N_D)} \)

- Do not forget the non-negativity constraints for the elements of \( y \)...
Getting the complete market-clearing

- By *complete* market-clearing is meant obtaining
  - list of offers (supply and demand) accepted and *their level*, as well as
  - the price at which the market is cleared, i.e., the so-called *market-clearing* or *system* price (in, e.g., Nord Pool)

- The system price is obtained through the *dual of the LP* defined before:

\[
\begin{align*}
\max_{\lambda, \nu} & \quad -b^T \nu \\
\text{subject to} & \quad A_{eq} \lambda - A^T \nu \leq c \\
& \quad \nu \geq 0
\end{align*}
\]

- This is also an LP: it can be solved with Matlab, R, GAMS, etc.

- \(\lambda\) and \(\nu\) are sets of *Lagrange multipliers* associated to all equality and inequality constraints:

\[
\lambda = \lambda^S \\
\nu = [\nu_1^G \ldots \nu_{NG}^G \nu_1^D \ldots \nu_{ND}^D]^T
\]

[Note: basics of optimization for application in electricity markets are given in:
More specifically for the market-clearing problem

- Only one equality constraint:

\[ \sum_i y^D_i - \sum_j y^G_j = 0 \]

for which the associated Lagrange multiplier \( \lambda^S \) represents the system price.

- And \( N_D + N_G \) inequality constraints:

\[ 0 \leq y^D_i \leq P^D_i, \quad i = 1, \ldots, N_D \]
\[ 0 \leq y^G_j \leq P^G_j, \quad j = 1, \ldots, N_G \]

for which the associated Lagrange multipliers \( \nu^D_i \) and \( \nu^G_j \) represents the unitary benefits for the various demand and supply offers if the market is cleared at \( \lambda^S \).

- The resulting LP writes:

\[
\begin{align*}
\max_{\lambda^S, \{\nu^D_i\}, \{\nu^G_j\}} & \quad - \sum_j \nu^G_j P^G_j - \sum_i \nu^D_i P^D_i \\
\text{subject to} & \quad \lambda^S - \nu^G_j \leq \lambda^G_j, \quad j = 1, \ldots, N_G \\
& \quad -\lambda^S - \nu^D_i \leq -\lambda^D_i, \quad i = 1, \ldots, N_D \\
& \quad \nu^G_j \geq 0, \quad j = 1, \ldots, N_G, \quad \nu^D_i \geq 0, \quad i = 1, \ldots, N_D
\end{align*}
\]

[To retrieve the dual LP, one may want to follow the steps described in: S Lahaie (2008) How to take the Dual of a Linear Program. (link)]
Let's also write it as a compact linear program!

- As for the **primal LP** allowing to obtain the dispatch for market participants on both supply and demand side, we write here the **dual LP** in a compact form:

  \[
  \max_{\tilde{y}} \quad \tilde{c}^T \tilde{y} \\
  \text{subject to} \quad \tilde{A} \tilde{y} \leq \tilde{b} \\
  \tilde{y} \geq 0
  \]

- The next 2 slides describe how to build the assemble the relevant vectors and matrices in the above LP...

- Then, it can be solved with **Matlab**, **R**, **GAMS**, etc.

- And, the solution \( \tilde{y}^* \) will give you the unit benefits for each and every market participant, as well as the equilibrium price...

[NB: Most optimization functions and tools readily give you the solution of dual problems when solving the primal ones! E.g., see documentation of linprog in Matlab]
The vector $\mathbf{y}$ of optimization variables $\mathbf{c}$ of weights in the objective function are constructed as

$$
\tilde{\mathbf{y}} = \begin{bmatrix}
\nu_1^G \\
\nu_2^G \\
\vdots \\
\nu_{N_G}^G \\
\nu_1^D \\
\nu_2^D \\
\vdots \\
\nu_{N_D}^D \\
\lambda_S
\end{bmatrix}, \quad \tilde{\mathbf{y}} \in \mathbb{R}^{(N_G+N_D+1)}
$$

$$
\tilde{\mathbf{c}} = \begin{bmatrix}
-P_1^G \\
-P_2^G \\
\vdots \\
-P_{N_G}^G \\
-P_1^D \\
-P_2^D \\
\vdots \\
-P_{N_D}^D \\
0
\end{bmatrix}, \quad \tilde{\mathbf{c}} \in \mathbb{R}^{(N_G+N_D+1)}
$$
Vector and matrices defining constraints

- No equality constraint!
- For the inequality constraint:

\[
\tilde{A} = \begin{bmatrix}
-1 & \cdot & \cdot & \cdot & \cdot & 0 & 1 \\
\cdot & -1 & \cdot & \cdot & \cdot & -1 & -1 \\
\cdot & \cdot & -1 & \cdot & \cdot & -1 & -1 \\
0 & \cdot & \cdot & \cdot & \cdot & -1 & -1
\end{bmatrix},
\tilde{b} = \begin{bmatrix}
\lambda_1^G \\
\lambda_2^G \\
\cdot \\
\lambda_{NG}^G \\
\cdot \\
-\lambda_1^D \\
-\lambda_2^D \\
\cdot \\
-\lambda_{ND}^D
\end{bmatrix},
\]

with \( \tilde{A} \in \mathbb{R}^{(N_G+N_D) \times (N_G+N_D)} \) and \( \tilde{b} \in \mathbb{R}^{(N_G+N_D)} \)
Application to our simple auction example

- Solving the **primal LP** for selecting the supply and demand offers yields:
  - Total Energy: 995 MWh
  - Supply side - accepted: \{G_1, \ldots, G_8\} (but only 55 MWh for G_8)
  - Supply side - rejected: \{G_9, \ldots, G_{15}\}
  - Demand side - accepted: \{D_1, \ldots, D_9\}
  - Demand side - rejected: \{D_{10}, \ldots, D_{12}\}

- Solving the **dual LP** gives:
  - the system price: 37.5 €/MWh
  - Unitary benefits on the demand and supply sides:

<table>
<thead>
<tr>
<th>Supply id.</th>
<th>Unitary benefit (€/MWh)</th>
<th>Demand id.</th>
<th>Unitary benefit (€/MWh)</th>
</tr>
</thead>
<tbody>
<tr>
<td>G_1</td>
<td>37.5</td>
<td>D_1</td>
<td>162.5</td>
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<tr>
<td>G_2</td>
<td>37.5</td>
<td>D_2</td>
<td>72.5</td>
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<td>G_3</td>
<td>22.5</td>
<td>D_3</td>
<td>62.5</td>
</tr>
<tr>
<td>G_4</td>
<td>7.5</td>
<td>D_4</td>
<td>52.5</td>
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<tr>
<td>G_5</td>
<td>5</td>
<td>D_5</td>
<td>47.5</td>
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<td>G_6</td>
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<td>37.5</td>
</tr>
<tr>
<td>G_7</td>
<td>1.5</td>
<td>D_7</td>
<td>27.5</td>
</tr>
<tr>
<td>G_8</td>
<td>0</td>
<td>D_8</td>
<td>2.5</td>
</tr>
<tr>
<td></td>
<td></td>
<td>D_9</td>
<td>0.5</td>
</tr>
</tbody>
</table>
Settlement process

After offers are selected and the system price determined, comes the settlement process...

In short:

- who should pay what?
- who should should get paid, and what amount?

Any opinion?
Settlement: pay-as-bid

- The first settlement option is the so-called **pay-as-bid** one.

- Method:
  
  - **Consumption side**: if $\lambda_i^D \geq \lambda^S$, consumer $i$ pays $\lambda_i^D$ for any unit of energy to be consumed
  
  - **Supply side**: if $\lambda_j^G \leq \lambda^S$, supplier $j$ is paid $\lambda_j^D$ for any unit of energy produced

---

**Example (continued):**

- **Consumption side**:
  
  - $D_1$ pays $250 \times 200 = 50,000\text{€}$
  
  - $D_2$ pays $300 \times 110 = 33,000\text{€}$
  
  - $D_3$ pays $120 \times 100 = 12,000\text{€}$, etc.

- **Supply side**:
  
  - $G_1$ receives $120 \times 0 = 0\text{€}(!!)$
  
  - $G_2$ receives $50 \times 0 = 0\text{€}(!!)$
  
  - $G_3$ receives $200 \times 15 = 3,000\text{€}$, etc.

- *Does that look like a good system?*
Settlement: pay-as-bid

- The first settlement option is the so-called **pay-as-bid** one.

- **Method:**
  - **Consumption side:** if $\lambda_i^D \geq \lambda^S$, consumer $i$ pays $\lambda_i^D$ for any unit of energy to be consumed
  - **Supply side:** if $\lambda_j^G \leq \lambda^S$, supplier $j$ is paid $\lambda_j^D$ for any unit of energy produced

---

Example (continued):

- **Consumption side:**
  - $D_1$ pays $250 \times 200 = 50,000\,€$
  - $D_2$ pays $300 \times 110 = 33,000\,€$
  - $D_3$ pays $120 \times 100 = 12,000\,€$, etc.

- **Supply side:**
  - $G_1$ receives $120 \times 0 = 0\,€(!!)$
  - $G_2$ receives $50 \times 0 = 0\,€(!!)$
  - $G_3$ receives $200 \times 15 = 3,000\,€$, etc.

- **Does that look like a good system?**
- Let’s play a game then... to show what can go wrong.
Settlement: uniform pricing

- The second settlement option is the so-called uniform pricing one.

- Method:
  - Consumption side: if $\lambda_i^D \geq \lambda^S$, consumer $i$ pays $\lambda^S$ for any unit of energy to be consumed
  - Supply side: if $\lambda_j^G \leq \lambda^S$, supplier $j$ is paid $\lambda^S$ for any unit of energy produced

**Example (continued):**

- Consumption side:
  - $D_1$ pays $250 \times 37.5 = 9.375\,€$
  - $D_2$ pays $300 \times 37.5 = 11.250\,€$
  - $D_3$ pays $120 \times 37.5 = 4.500\,€$, etc.

- Supply side:
  - $G_1$ receives $120 \times 37.5 = 4.500\,€(!!)$
  - $G_2$ receives $50 \times 37.5 = 1.875\,€(!!)$
  - $G_3$ receives $200 \times 37.5 = 7.500\,€$, etc.

- *Does that look like a better system?*
Network effects
The transmission “problem”

- Remember there is a network involved, and power has to flow...

- This was not represented so far in our market description!
Approaches to representing network constraints

There are basically two philosophies, developed on both sides of the Atlantic:

- US
- Europe

<table>
<thead>
<tr>
<th></th>
<th>Europe</th>
<th>US</th>
</tr>
</thead>
<tbody>
<tr>
<td>System Operator</td>
<td>TSO</td>
<td>ISO</td>
</tr>
<tr>
<td>Market Operator</td>
<td>Ind. Market Operator</td>
<td>ISO</td>
</tr>
<tr>
<td>Offers</td>
<td>Market products</td>
<td>Unit capabilities</td>
</tr>
<tr>
<td>Clearing</td>
<td>Supply-demand equilibrium</td>
<td>UCED problem</td>
</tr>
<tr>
<td>Prices</td>
<td>Zonal</td>
<td>Nodal</td>
</tr>
</tbody>
</table>

TSO: Transmission System Operator
ISO: Independent System Operator
UCED: Unit Commitment and Economic Dispatch
Illustration of Zonal and Nodal pricing

Scandinavia (Zonal):

Midwest US (Nodal):

Go visit: [http://nordpoolgroup.com](http://nordpoolgroup.com) (market data, map)

Go visit: [https://www.misoenergy.org](https://www.misoenergy.org)
From system to area prices

Let us revisit our previous example,

considering two areas: DTU-West and DTU-East

with a transmission capacity of 40 MW

Demand: (for a total of 1065 MWh)

<table>
<thead>
<tr>
<th>Company</th>
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<th>Amount (MWh)</th>
<th>Price (€/MWh)</th>
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</tr>
</thead>
<tbody>
<tr>
<td>CleanRetail</td>
<td>D1</td>
<td>250</td>
<td>200</td>
<td>DTU-West</td>
</tr>
<tr>
<td>El4You</td>
<td>D2</td>
<td>300</td>
<td>110</td>
<td>DTU-East</td>
</tr>
<tr>
<td>EVcharge</td>
<td>D3</td>
<td>120</td>
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<tr>
<td>QualiWatt</td>
<td>D4</td>
<td>80</td>
<td>90</td>
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</tr>
<tr>
<td>IntelliWatt</td>
<td>D5</td>
<td>40</td>
<td>85</td>
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<tr>
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<td>D12</td>
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Supply: (for a total of 1435 MWh)

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<th>Company</th>
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<th>Price (€/MWh)</th>
<th>Area</th>
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<tr>
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<td>200</td>
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</tr>
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</table>
Localizing the previous market-clearing results

Following previous market clearing results, one obtains

- **DTU-West:**
  - Supply side: \{G_1, G_3, G_5, G_7, G_8\} (but only 55 MWh for G_8) - Total: 495 MWh
  - Demand side: \{D_1, D_3, D_5, D_6, D_8, D_9\} - Total: 555 MWh
  → Deficit of 60 MWh

- **DTU-East:**
  - Supply side: \{G_2, G_4, G_6\} - Total: 500 MWh
  - Demand side: \{D_2, D_4, D_7\} - Total: 440 MWh
  → Surplus of 60 MWh

**BUT**, only 40 MWh can flow through the interconnection!
Market split: Import-Export approach

- Due to transmission constraints, the market has to split and becomes two markets

\[\text{DTU-West}\]
\[\text{DTU-East}\]

- In practice:
  - 2 market zones with their own supply-demand equilibrium
  - Extra (price-independent) consumption/generation offers representing the transmission from one zone to the next to be added
Adding transmission-related offers

- Extra supply in the high price area (40 MWh coming from DTU-East)

- Extra consumption in the low price area (40 MWh for DTU-West)

- Power ought to flow from a low price area to a high price area
Results for each zone

- The same type of LP problems as introduced before is solved
  - for each zone individually,
  - with the extra consumption/generation offers representing the amount of energy transmitted

- That eventually yields
  - DTU-West:
    - Supply side: \{G_1, G_3, G_5, G_7, G_8\} (but only 75 MWh for G_8) - Total: 515 MWh
    - Demand side: \{D_1, D_3, D_5, D_6, D_8, D_9\} - Total: 555 MWh
      \[\rightarrow\] Zonal price: 37.5 €
  - DTU-East:
    - Supply side: \{G_2, G_4, G_6\} (but only 30 MWh for G_6) - Total: 480 MWh
    - Demand side: \{D_2, D_4, D_7\} - Total: 440 MWh
      \[\rightarrow\] Zonal price: 34 €

A few questions at this stage:
- What is the impact on the settlement?
- Do you think it would generalize well for more than 2 zones?
Instead of boldly splitting the market, one could instead acknowledge how power flows...

- Our system with 2 zones can be modelled as a 2-bus system,
- Loads and generators are associated to the relevant bus
- DC power flow can be assumed to simplify things...
Formulating the market clearing

- The network-constrained social welfare maximization problem can be written as:

\[
\begin{align*}
\text{max} & \quad \sum_i \lambda_i^D y_i^D - \sum_j \lambda_j^G y_j^G \\
\text{subject to} & \quad \sum_i y_i^{D,\text{West}} - \sum_j y_j^{G,\text{West}} = B \Delta \delta \\
& \quad \sum_i y_i^{D,\text{East}} - \sum_j y_j^{G,\text{East}} = -B \Delta \delta \\
& \quad 0 \leq y_i^D \leq P_i^D, \quad i = 1, \ldots, N_D \\
& \quad 0 \leq y_j^G \leq P_j^G, \quad j = 1, \ldots, N_G \\
& \quad -40 \leq B \Delta \delta \leq 40
\end{align*}
\]

where:
- \(B\) is the absolute value of susceptance (physical constant) of the interconnection between DTU-West and DTU-East
- \(\Delta \delta\) is the difference of voltage angles between the 2 buses
  \(\rightarrow B \Delta \delta\) represents the signed power flow from DTU-West to DTU-East
Obtaining the zonal prices

- As for the case of a single zone, the dual LP allows to obtains market-clearing prices.
- These 2 prices corresponds to the Lagrange multipliers for the 2 equality constraints (i.e., balance equations):

\[
\max_{\{y^D_i\},\{y^G_j\}} \sum_i \lambda^D_i y^D_i - \sum_j \lambda^G_j y^G_j
\]

subject to

\[
\sum_i y^D_i,\text{West} - \sum_j y^G_j,\text{West} = B\Delta \delta : \lambda^S,\text{West}
\]

\[
\sum_i y^D_i,\text{East} - \sum_j y^G_j,\text{East} = -B\Delta \delta : \lambda^S,\text{East}
\]

\[
0 \leq y^D_i \leq P^D_i, \quad i = 1, \ldots, N_D
\]

\[
0 \leq y^G_j \leq P^G_j, \quad j = 1, \ldots, N_G
\]

\[-40 \leq B\Delta \delta \leq 40\]
Results for our auction example

- For the 2 zones...
  - DTU-West:
    - Supply side: \( \{ G_1, G_3, G_5, G_7, G_8 \} \) (but only 75 MWh for \( G_8 \)) - Total: 515 MWh
    - Demand side: \( \{ D_1, D_3, D_5, D_6, D_8, D_9 \} \) - Total: 555 MWh
      \[ \rightarrow \text{Zonal price: } 37.5 \, \text{€} \]
  - DTU-East:
    - Supply side: \( \{ G_2, G_4, G_6 \} \) (but only 30 MWh for \( G_6 \)) - Total: 480 MWh
    - Demand side: \( \{ D_2, D_4, D_7 \} \) - Total: 440 MWh
      \[ \rightarrow \text{Zonal price: } 34 \, \text{€} \]

- The dispatch and prices are the same as before, though...
  - one relies on a rigorous optimize problem that acknowledge how power flows on the network,
  - it should readily scale to more than 2 zones
Final extension to nodal pricing

- In a US-like setup, each node of the power system is to be seen as an area...
- For a system with $K$ nodes, the network-constrained social welfare maximization market-clearing writes:

$$\max \{\lambda_i^D y_i^D, \lambda_j^G y_j^G\}$$

subject to

$$\sum_i y_i^{D,k} - \sum_j y_j^{G,k} = \sum_{l \in \mathcal{L}_k} B_{kl}(\delta_k - \delta_l), \ k = 1, \ldots, K : \lambda^S,k$$

$$0 \leq y_i^D \leq P_i^D, \ i = 1, \ldots, N_D$$

$$0 \leq y_j^G \leq P_j^G, \ j = 1, \ldots, N_G$$

$$-C_{kl} \leq B_{kl}(\delta_k - \delta_l) \leq C_{kl}, \ k, l \in \mathcal{L}_N$$

where

- $\mathcal{L}_N$ is the set of nodes, $\mathcal{L}_k$ the set of nodes connected to node $k$
- $B_{kl}$ are the line susceptances, $(\delta_k - \delta_l)$ the phase angle differences
- $\lambda^S,k$ are the $K$ nodal prices

[More on Locational Marginal Pricing: Enerdynamics (2012). Locational marginal Pricing. Electricity Markets Dynamics online course (video - 3’26 mins)]

31761 - Renewables in Electricity Markets
Different types of market products (a quick tour...)
From simple to more advanced products

- So far, we only considered the most basic market product, i.e., a quantity of energy for a single time unit.
- An electricity market based on such a simple market product may not function very well...

why?
From simple to more advanced products

- So far, we only considered the most basic market product, i.e., a quantity of energy for a single time unit.

- An electricity market based on such a simple market product may not function very well...

**why?**

Since operational constraints, and flexibility, need to be somewhat represented!!

- These may include
  - minimum and maximum generation levels
  - minimum up and down time
  - ramping constraints
  - varying costs
  - flexibility in generation and consumption
  - etc.
Some of the more advanced products (1)

- Stepwise offer curves:
  - Different potential energy blocks at different prices
  - Can reflect for (e.g., supply side)
    - minimum and maximum generation levels
    - varying generation costs
  - Also relevant on the demand side

- It is straightforward to extend market-clearing algorithms… do you see how?
Some of the more advanced products (2)

- **Block offer:** Let me offer to generate/consume $X$ MWh, uniformly distributed over $N$ consecutive time units

  → Allow to account for: (i) minimum up/down time; and (ii) minimum/maximum generation levels and varying costs to a lesser extent

- **Flexible offer:** Let me offer to generate/consume $X$ MWh over any single time unit of the day

  → Allow to account for: flexibility in generation and consumption (e.g., industrial processes)

- **It is NOT** straightforward to extend market-clearing algorithms... *do you see how?*
Some of the more advanced products (2)

- **Block offer:** Let me offer to generate/consume \(X\) MWh, uniformly distributed over \(N\) consecutive time units
  
  \[ \rightarrow \text{Allow to account for: (i) minimum up/down time; and (ii) minimum/maximum generation levels and varying costs to a lesser extent} \]

- **Flexible offer:** Let me offer to generate/consume \(X\) MWh over any single time unit of the day
  
  \[ \rightarrow \text{Allow to account for: flexibility in generation and consumption (e.g., industrial processes)} \]

- **It is NOT straightforward to extend market-clearing algorithms... do you see how?**

- **This is since:**
  
  - all market time units have to be considered altogether
  
  - integer variables have to used for the various on/off states
  
  \[ \rightarrow \text{one needs to formulate a Mixed Integer Linear Program (MILP)} \]
Questions / discussion
For you to do...

Before the next session on Monday 5 February 2018

For those who want to know more about wholesale markets:


For those who need a refresh on linear programming:

- Optimization Methods in Management Science / Operations Research (2013). *Introduction to LP formulations*. MIT OpenCourseWare (link to slides)

For those who need a refresh on linear programming and duality:

- Thomas S. Ferguson (????). *Linear Programming - A Concise Introduction*. Electronic text
Large-scale integration of renewable energy

Books

In an effort to disseminate our work to students, researchers and practitioners, some collaborators and I have been focusing on producing books that would gather knowledge in renewable energy, forecasting, and electricity markets. For a description of these books, press the links "Electricity markets" and "Forecasting" under the header "Books".

Wind power forecasting

It is not possible to decide on the level of wind energy to be produced in the coming minutes or days – one relies on nature and the weather. Ways have to be found to optimally assimilate this energy generation in the system. Wind power modeling and forecasting is recognized as a cost-effective and necessary solution to that problem. In my research, I have been looking at a few aspects of wind power forecasting, which I rapidly describe here...

A little toy...

If you wonder how future renewable energy forecasting may look, let me invite you to look at this toy forecasting system, which we will make evolve as new features are to become available.

Read more »
Appendix

- For a fixed and inflexible demand $D$, the market-clearing becomes:

$$\min \{y_j^G\} \quad \sum_j \lambda_j^G y_j^G$$
subject to \quad \sum_j y_j^G = D

$$0 \leq y_j^G \leq p_j^G, \quad j = 1, \ldots, N_G$$

- Though, since overall supply might happen to be less than demand, one adds a component reflecting the value of lost load $\tau$, for the part of the load $\delta D$ not served:

$$\min \{y_j^G, \delta D\} \quad \left(\sum_j \lambda_j^G y_j^G\right) + \tau \delta D$$
subject to \quad \sum_j y_j^G + \delta D = D

$$0 \leq y_j^G \leq p_j^G, \quad j = 1, \ldots, N_G$$
$$0 \leq \delta D \leq D$$

$\tau$ is usually set to a high value, e.g., $\tau = 1000\,\text{€/MWh}$